GRAVITATION

by (in part)

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conclusions can be given concerning the proportion of this activity caused by invertebrates and other decomposing reducers, since it has not yet been quantitatively evaluated. Under some conditions such organisms are very numerous. The liveweight of termites in the tropical savanna of the Ivory Coast, for example, has been found to be 40 grams per square metre in an area where aboveground herbivores measure only one gram per square metre. It is generally agreed, however, that the soil microflora (microscopic plants, including fungi and bacteria) is the single most important group of organisms affecting the turnover of energy. The biomass of soil microflora has been estimated to be 400 to 600 grams per square metre of soil surface on a dry-weight basis; however, probably less than 1 percent of this is active at any one time.

Importance of decomposer organisms

It is not yet possible to identify to what extent various groups of soil microflora contribute to the total of decomposer activity in any given system. The rate of decomposition in standing dead vegetation is usually slower, however, than in litter and in vegetation in contact with the ground. The bulk of the microbial decomposition does not occur until the litter has either made contact with the soil or has become densely compressed just above the soil surface. The smaller macrofauna (animals bigger than microbes), such as litter-feeding insects and worms, are active in bringing litter into more intimate contact with the soil.

UTILIZATION OF GRASSLANDS

Many early human civilizations developed in grassland regions, so man should be familiar with the ecology of grasslands. Greater interest, however, has been shown in converting grasslands to the growth of annual crops than has been devoted to considering whether they would have been a more valuable resource in an untilled state.

Domesticated grazing animals occupy the most important role in man's conversion of natural-grassland plant growth to a form of food that satisfied him. While conservationists in the past may have looked upon the domesticated grazing animal as an intruder in natural grassland, this does not necessarily mean that the grassland environment will deteriorate through the replacement of natural by introduced animals. The philosophy of range management that has developed in North America is based on the concept of obtaining the highest sustained level of animal production on natural grassland that is compatible with maintenance of the resource. Range ecologists have been much more conscious of the need to conserve land resources than have agriculturalists. In the management of arable lands, for example, the guiding principle has been almost exclusively determined by the need to produce the maximum harvestable yield, a practice hardly compatible with conservation. The advantage that the ecologist sees in the use of domesticated livestock in the rational management of rangelands is that the distribution and density of these animals is under his control to a far greater extent than would be possible with native animals. The domestication of the range is thus seen as a stabilizing situation.

The success achieved in increasing the harvestable yield of intensively managed arable land and improving semipermanent grasslands of woodland climate has led to the suggestion that the plant cover of nonarable grasslands should be changed as much as possible by the introduction of domesticated forage crops that have been selected and bred for high yield and for optimum response to fertilization and management. A high degree of environmental control is needed, however, to utilize the higher potential of these species, and the indication is that the native grass cover cannot be excelled by introduced species for range production on nonarable land.

Effects of herbicides and fertilizers

Attempts have been made to increase the productivity of natural grassland by the use of herbicides and fertilizers. Weed control of rangeland is economically practical only in cases where the weedy situation has been induced by mismanagement and when this situation is corrected. The effect of fertilizers is variable, yielding better returns where moisture conditions are most favourable. Some native species do not respond to increased levels of nutrient supply and may be replaced after fertilization by species that are more productive but dependent for survival on a continued supply of fertilizer nutrients.

After the initiation of tillage, highly fertile, temperate grasslands gradually (over a period of 50 to 100 years) attain a new level of equilibrium, which is associated with a lower content of soil organic matter. The full impact on organic-matter content will not occur until the original organic material is replaced by that formed from the annual agricultural plant cover. The concept that corrective measures can be taken by future human generations by addition of chemical fertilizer does not account for changes in soil structure that may be associated with declining organic content.

The maintenance of both arable and nonarable ecosystems in grassland zones is vital to the continued provision of food for the world. The temperate grassland zones include a very important portion of the cropland of the earth (for example, 90 percent of the grain for commerce originates here); the tropical and subtropical grasslands and savannas provide a possible means for expanding agriculture when technology is developed to manage these lands on a long-term basis.

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(R.T.C.)

Gravitation

Gravitation is a universal force of attraction acting between all matter. The trajectories of bodies in the solar system are determined, except for relatively tiny effects, by the laws of gravity, while on Earth all bodies have a weight or downward force of gravity proportional to their mass, which the Earth's mass exerts on them. Gravity is by far the weakest known force in nature and therefore plays no role in determining the internal properties of everyday matter. But because of the long reach and universality of the gravitational attraction, gravity plays a central role in shaping the structure and evolution of stars, galaxies, and the entire universe.

The works of Isaac Newton and Albert Einstein dominate the development of gravitational theory. Newton's classical theory of gravitational force held sway from its presentation in his *Principia*, published in 1687, until Einstein's work in the early 20th century. Even today, Newton's theory is of sufficient accuracy for all but the most precise applications. Einstein's modern field theory of general relativity predicts only minute quantitative differences from the Newtonian theory except in a few special cases. The major significance of Einstein's theory is its radical conceptual departure from classical theory and its implications for further growth in physical thought (see RELATIVITY).

DEVELOPMENT OF GRAVITATIONAL THEORY

Early concepts. The classical Greek philosophers considered the motions of the celestial bodies and of objects on Earth as basically unrelated. The former was not considered as gravitationally determined, as the celestial bodies were seen to follow perpetually repeating, nondescending trajectories in the sky. Aristotle envisioned such bodies as possessing "natural" motions that did not require external causes or agents. In this view, celestial bodies underwent their own particular "natural" motion, while massive earthly objects possessed a natural tendency to move toward the Earth's centre. Two other Aristotelian viewpoints were: that a body moving at constant speed required a continuous force acting on it and that force must be applied by contact rather than interaction or force at a distance. These views impeded understanding of the principles of motion and hence retarded the development of a theory of universal gravitation. During the 16th and early 17th century, however, several scientific contributions to the problem of earthly and celestial motion set the stage for Newton's gravitational theory.

Kepler's laws of planetary motion

Johannes Kepler, accepting the Copernican perspective —in which the planets orbited the Sun rather than the Earth—and using Tyco Brahe's improved measurements of planetary movements, succeeded in describing the planetary orbits by simple geometrical and arithmetical relations. Kepler's three quantitative laws of planetary motion were: (1) the planets describe elliptic orbits, of which the Sun occupies one focus; (2) the line joining a planet to the Sun sweeps out equal areas in equal time; and (3) the square of the period of revolution of a planet is proportional to the cube of its average distance from the Sun. During this same period, Galileo made major progress in understanding the properties of "natural" motion and simple accelerated motion for earthly objects. He realized that bodies uninfluenced by forces would continue indefinitely to move and that force was necessary to change motion, not to maintain constant motion. Galileo performed experiments to show that the Earth's gravity produced constant downward acceleration and that the downward gravitational acceleration was independent of the bulk or composition of bodies.

Newton's law of gravity. The modern quantitative science of gravitation began with the work of Newton. He assumed the presence of an attractive force between all massive bodies; this force does not require bodily contact but acts at a distance. By invoking his law of inertia (bodies not acted upon by a force move at constant speed in a straight line), Newton concluded that a gravitational force exerted by the Earth on the Moon was needed to keep it in a circular motion about the Earth rather than in a straight line and that this force could be, at long-range, of the same kind as the force with which the Earth pulled objects on its surface downward. Galileo had previously measured the downward acceleration of bodies on Earth to be approximately 980 centimetres (32 feet) per second per second, while Newton calculated that circular orbital motion of radius R and period T required a constant inward acceleration A equal to the product of $4\pi^2$ and the ratio of the radius to the square of the time:

$$A = \frac{4\pi^2 R}{T^2}. \qquad (1)$$

Applying this formula to the Moon's orbit, which has a radius of about 384,000 kilometres (about 60 Earth radii) and a period of 27.3 days, the inward acceleration is approximately 2.7×10^{-3} metres per second per second; this is the same as 1/3,600 ($1/60^2$) times the Earth's surface acceleration. Newton deduced that the gravitational force between bodies diminishes as the inverse square of the distance between the bodies, as he could thus relate the two accelerations to a common interaction. A further assumption, that the mass of the Earth acts gravitationally on the outside world as if the mass were concentrated at the Earth's centre, was needed to obtain his relationship. Newton proved mathematically that this assumption was true for all spherically symmetric bodies.

Newton saw that the gravitational force between bodies must depend on masses of the bodies. Since a body of mass M experiencing a force F accelerates at a rate F/M, a force of gravity proportional to M would be consistent with Galileo's observation that all bodies accelerate under gravity toward Earth at the same rate. Newton's law of gravity can therefore be expressed mathematically by a relation expressing the law. If F_{12} is the magnitude of the gravitational force acting between masses M_1 and M_2 separated by distance D_{12}, then the force equals the product of these masses and of G, a universal constant divided by the square of the distance, D_{12}^2:

The universal constant G

$$F_{12} = \frac{GM_1M_2}{D_{12}^2}. \qquad (2)$$

The constant G is a quantity having the physical dimensions $(\text{length})^3/(\text{mass})(\text{time})^2$, its numerical value depending on the physical units of mass, length, and time used. (G is discussed more fully in a later section.) To obtain the total gravitational force on a body produced by many masses represented as M_i, where the subscript i stands for the positions 1, 2, . . . n, the individual forces must be vectorially added together, as force has direction as well as magnitude. Letting $\mathbf{D}_i$ be the spatial vector from the mass M to the mass M_i in the same way, the force on M due to several masses becomes the sum of the forces due to each mass separately. In the following expression, in addition to the quantities already explained, the symbol Σ represents the sum, and the factor $\mathbf{D}_i/D_i^3$ is needed to give the direction and numerically is equivalent to division by D_i^2:

$$\mathbf{F} = GM \sum_{i=1}^{n} \frac{M_i \mathbf{D}_i}{D_i^3}. \qquad (3)$$

This is Newton's gravitational law in its vector form. The simpler expression gives the surface acceleration on Earth; setting a mass equal to the Earth's mass M_E and the distance equal to the Earth's radius r_E, the downward acceleration of a body at the surface g is equal to the product of the universal gravitational constant and the mass of the Earth divided by the square of the radius:

$$g = \frac{GM_E}{r_E^2}. \qquad (4)$$

Weight and mass

The weight W of the body can be measured by the equal and opposite force necessary to prevent the downward acceleration; this is Mg. The same body placed on the Moon's surface has the same mass, but as the Moon has a mass of about 1/81 times that of the Earth and a radius of only 0.27 of that of the Earth, the body on the Moon's surface acquires a weight of only 1/6 its Earth weight, as demonstrated by the U.S. Apollo astronauts. In orbiting satellites, where no force prevents the free fall of the satellites in the gravitational field, the cargo of humans and instruments experiences weightless conditions although the masses remain the same as on Earth.

The two equations above can be used to derive Kepler's third law, for the case of circular planetary orbits. By putting the expression for the acceleration A in equation (1) equal to the force of gravity for the planet, GM_PM_S/R^2, divided by the planet's mass M_P, M_S being the mass of the Sun, and R, T being the radius and period of the orbit, respectively, the following equation is obtained:

$$\frac{GM_S}{R^2} = \frac{4\pi^2 R}{T^2}$$

or

$$R^3 = \left(\frac{GM_S}{4\pi^2}\right) T^2. \qquad (5)$$

Newton was able to show that all three of Kepler's observationally derived laws followed mathematically from the assumption of his own laws of motion and the law of gravity stated above. In all observations of the motion of a celestial body, only the product of G and the mass can be found. Newton first estimated the magnitude of G by assuming the Earth's average mass density to be about 5.5 that of water, somewhat greater than the Earth's surface rock density, and calculating the Earth's mass from this. Then, taking M_E and r_E as the Earth's mass and radius, respectively, the value of G was

$$G = \frac{gr_E^2}{M_E}, \qquad (6)$$

which numerically comes close to the accepted value of

6.7×10^{-8} (cm)^3/(gm)(sec)^2, first directly measured by a Cavendish balance experiment (see below).

Comparing equation (4) above for the Earth's surface acceleration g with the R^3/T^2 ratio for the planets, a formula for the ratio of the Sun's mass, M_S, to Earth's mass, M_E, was obtained in terms of known quantities, R_E being the radius of the Earth's orbit:

$$\frac{M_S}{M_E} = \frac{4\pi^2 R_E^3}{g T_E^2 r_E^2} \cong 325{,}000\,. \qquad (7)$$

By using observations of the motion of the moons of Jupiter discovered by Galileo, Newton determined that Jupiter was 318 times more massive than Earth but only ¼ as dense, having a radius 11 times larger than Earth.

Inter-action among celestial bodies

When two celestial bodies of comparable mass interact gravitationally, the bodies each orbit about a fixed point (the centre of mass of the two bodies), which lies between the bodies on the line joining them at a position such that the distances to each body multiplied by each body's mass are equal. Observing that the Sun's apparent position in the ecliptic (the plane in which the Sun seems to be moving around the Earth) oscillates every month by about 12 arc-seconds (superimposed upon its annual motion) and accounting for this by means of a motion of the Earth around the Earth–Moon centre of mass, it was concluded that this centre of mass is placed about 4,800 kilometres (3,000 miles) toward the Moon from the Earth centre. From this, the Moon was found to be about 1/81 (4,800/384,000) as massive as the Earth. With slight modifications Kepler's laws remain valid for systems of two comparable masses; the foci of the elliptical orbits are the two-body centre-of-mass positions, and putting $M_1 + M_2$ instead of M_S in the expression of Kepler's third law, equation (5) above, the third law reads:

$$R^3 = \frac{G(M_1 + M_2)}{4\pi^2} T^2. \qquad (8)$$

This agrees with equation (5) when one body is so small that its mass can be neglected. The rescaled formula can be used to determine the separate masses of binary stars (pairs of stars orbiting around each other; see STAR) that are of a known distance from the solar system. Equation (8) determines the sum of the stars' masses; and, if R_1, R_2 are the distances of the individual stars from the centre of mass, the ratio of the distances must balance the inverse ratio of the masses, and the sum of the distances is the total distance R. In symbols,

$$\frac{R_1}{R_2} = \frac{M_2}{M_1};\ R_1 + R_2 = R. \qquad (9)$$

These relations are sufficient to determine the individual masses. Observations of the orbital motion of double stars, of the dynamical motion of stars collectively moving within their galaxies, and of the motion of the galaxies themselves verify that Newton's law of gravity is valid to a high degree of accuracy throughout the visible universe.

Ocean tides, phenomena that mystified thinkers for centuries, were also shown by Newton to be a consequence of the universal law of gravitation, although the details of the complicated phenomena were not understood until comparatively recently. They are caused specifically by the gravitational pull of the Moon and, to a lesser extent, of the Sun (see TIDES).

The discovery of Neptune

In the period following Kepler and Newton, improved accuracy in the measurements of planetary motion led to small discrepancies from the simple predictions of Kepler's laws. All but a few were later shown in accord with the universal aspect of Newton's law of gravity. Small corrections due to the fact that all the planets must perturb each other explained almost all variations in the planets' motions. The exceptions proved to be of large importance. Uranus, the seventh planet from the Sun, was observed to undergo variations in its motion that could not be explained by perturbations due to Saturn, Jupiter, and the other planets. The English mathematician John Couch Adams and the French mathematician Urbain-Jean-Joseph Le Verrier independently assumed the presence of an unseen eighth planet that could produce the observed discrepancies in the motion of Uranus. They calculated its position within a degree of which the planet Neptune was discovered in 1846. Measurements of the motion of the innermost planet, Mercury, over an extended period led astronomers to conclude that the major axis of this planet's elliptical orbit precessed (precession is the gyration or wobble of the axis of a rotating body affected by a gravitational field) in space at a rate 43 arc-seconds per century faster than could be accounted for from perturbations of the other planets. In this case, however, no other bodies could be found that could produce this discrepancy, and very slight modification of Newton's law of gravitation seemed to be needed. Einstein's theory of relativity precisely predicts this observed behaviour of Mercury's orbit (see RELATIVITY).

INTERPRETATION OF GRAVITY MEASUREMENTS

Potential theory. For irregular, nonspherical mass distributions in three dimensions, the vector equation (3) above, which expresses Newton's law of gravity essentially in its original form, is inefficient, though theoretically it could be used for finding the resulting gravitational field. The main progress, after Newton, in classical gravitation theory was the introduction of potential theory, which allows practical as well as theoretical investigation of the gravitational variations in space and anomalies due to the irregularities and shape deformations of the Earth.

Potential theory led to the following elegant formulation: The gravitational acceleration, g, a function of position $\boldsymbol{R}$, $g(\boldsymbol{R})$, at any point in space is given from a function, Φ, called the gravitational potential, by means of a generalization of the operation of differentiation:

$$g(\boldsymbol{R}) = \frac{\partial \Phi}{\partial x} \boldsymbol{i} + \frac{\partial \Phi}{\partial y} \boldsymbol{j} + \frac{\partial \Phi}{\partial z} \boldsymbol{k},$$

in which $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ stand for unit basis vectors in a three-dimensional Cartesian coordinate system. Whatever restriction on g is introduced by the mass density ρ, that restriction is then transferred to the potential function Φ and is expressed in an equation that was discovered by the French mathematician Siméon-Denis Poisson: Poisson's equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\Phi(\boldsymbol{R}) = -4\pi G \rho(\boldsymbol{R}),$$

in which the equation is to hold for well specified values of $\boldsymbol{R}$. The significance of this approach is seen by observing that Poisson's equation can be solved under rather general conditions, which is not the case with Newton's equation. When ρ is non-zero, the solution is expressed as a definite integral:

$$\Phi(\boldsymbol{R}) = G \int \frac{\rho(\boldsymbol{R}')d\boldsymbol{R}'}{|\boldsymbol{R} - \boldsymbol{R}'|}\,.$$

When $\rho = 0$ (that is, outside the Earth), Poisson's equation reduces to the simpler equation of Laplace and has a general solution expressed as a series of powers of the trigonometric cosine function of θ, the latitude angle measure from the north pole:

$$\Phi(R) = \frac{GM_E}{R}\left[1 - J_2\left(\frac{R_E}{R}\right)^2 \frac{3\cos^2\theta - 1}{2} - J_3\left(\frac{R_E}{R}\right)^3 \frac{5\cos^3\theta - 3\cos\theta}{2} + \dots\right]$$

in which, if R is the distance from the Earth's centre, R_E is the average Earth radius, θ is the latitude angle measured from the north pole, and J_2, J_3, etc., are constants. The constants J_2, J_3, etc., are determined by the detailed mass distribution of the Earth; and, since Newton showed that for a spherical body all the J_n are absent, the J_n must be measurements of the deformation of the Earth from a spherical shape; J_2 measures the magnitude of the Earth's rotational equatorial bulge, J_3 measures a slight pear-shaped deformation of the Earth, and so on. By comparison of gravimeter measurements (see below) from many parts of the Earth's surface and by observations of perturbations on satellite orbits and the Moon's orbit, the parameters J_2 and J_3 have been found to be 1,082.7 $\times 10^{-6}$ and -2.4×10^{-6}, respectively. Higher terms in the potential series are being detected and measured by continuous observation of near-Earth satellite orbits.

Effects of local mass differences. The method used to describe global features of the Earth's mass distribution is inadequate to represent gravitational variations due to local mass differences such as mountain ranges, mineral deposits of unusual density, ocean basins, etc.; so other methods have been developed for these purposes. Several geological features of the Earth were first discovered from gravity measurements. Using gravimeters and horizontal pendulums, observers expected the additional bulk of mountain ranges compared with surrounding plains to produce slight attractive gravitational forces; instead, in many cases, slight repulsion was found. Such repulsion supports the view that mountain ranges float, possessing deep roots, much as an iceberg does, of underlying lightweight material that displaces the denser material of the Earth's interior. Portable gravimeters, which can detect variations of a part in 10^9 in the gravitational force, are today in wide use for mineral and oil prospecting, unusual underground deposits revealing their presence by creating local gravitational variations (see below).

Mascons

Man-made satellites tracked during orbital motion around the Moon and around the planet Mars have followed trajectories that are best understood by assuming that these bodies possess large regions of anomalous mass densities (called mascons) that produce gravitational field variations. The lunar mascons are of sufficient size to have perturbed the Apollo manned lunar landings, and the paths of the approaching capsules were found to need adjustment to account for the perturbations and to reach their desired landing sites.

GRAVITATIONAL THEORY AND OTHER ASPECTS OF PHYSICAL THEORY

Special relativity and electromagnetic field theory

The Newtonian theory of gravity is based on an assumed force acting between all pairs of bodies; that is, an action at a distance. When a mass moves, the force acting on other masses has been considered to adjust instantaneously to the new location of the displaced mass. Special relativity theory states that all physical signals travel no faster than the speed of light. This theory, with the field theory of electrical and magnetic phenomena, have met such empirical success, however, that most modern gravitational theories are constructed as field theories consistent with the principles of special relativity. In a field theory the gravitational force between bodies is formed by a two-step process: (1) One body produces a gravitational field that permeates all surrounding space but has weaker strength farther from its source. A second body in that space is then acted upon by this field, experiencing a force. (2) The Newtonian force of reaction is then viewed as the response of the first body to the gravitational field produced by the second body, there being at all points in space a superposition of gravitational fields due to all the bodies in it.

Field theories of gravitation. If the gravitational field has a theoretical and conceptual existence of its own, various new predictions of gravitational phenomena can be made. The equations governing the time evolution of the field predict a finite speed of propagation of disturbances through the field, replacing the direct instantaneous action at a distance by a delayed interaction transmitted by the field. The gravitational field in most modern theories is, in fact, free to change dynamically in certain modes independent of sources and to transmit energy and momentum in these modes, in a manner similar to electromagnetic wave radiation. These gravitational waves, which have been extensively studied mathematically since Einstein first showed their existence in theory, may have recently been demonstrated experimentally in the form of pulsed radiation received from the centre of the Galaxy. The possible detection of these extra-terrestrial gravitational waves was performed by monitoring cotemporaneous mechanical vibrations present in a pair of isolated, several-ton aluminum cylinders located about 1,500 kilometres (900 miles) apart. Field theories of gravity predict that gravity waves will excite such mechanical oscillations in these rigid bodies.

Corrections of Newtonian force law

Field theories of gravity, Einstein's general relativity being an important example, also predict specific corrections to the Newtonian force law, the corrections being of two basic forms: (1) When matter is in motion, additional gravitational fields (analogous to the magnetic fields produced by moving electric charges) are produced; also, moving bodies interact with gravitational fields in a motion-dependent way. (2) Unlike electromagnetic field theory, in which two or more electric or magnetic fields superimpose by simple addition to give the total fields, in gravitational field theory nonlinear fields proportional to the second and higher power of the source masses are generated, and gravitational fields proportional to the product of different masses are created. Gravitational fields themselves become sources for additional gravitational fields. Examples of some of these effects are shown below. The acceleration, **A**, of a moving particle of negligible mass that interacts with a mass, *m*, which is at rest, is given, in the following formula, derived from Einstein's gravitational theory. The expression for **A** now has, as well as the Newtonian expression as given in equation (1), further terms in higher powers of Gm/R^2—that is, in G^2m^2/R^4— and **V** is the particle's velocity vector, **A** its acceleration vector, **R** the vector from the mass *m*, and *c* is the speed of light. When written out, the sum is

$$\mathbf{A} = -\frac{Gm\mathbf{R}}{R^3} + 2\frac{G^2m^2\mathbf{R}}{c^2R^4} - \frac{3}{2}\frac{Gm\mathbf{R}}{R^3}\left(\frac{V^2}{c^2}\right) - \frac{\mathbf{V}\cdot\mathbf{AV}}{c^2} - \tfrac{1}{2}\frac{V^2}{c^2}\mathbf{A} + \dots$$

This expression gives only the first post-Newtonian corrections; terms of higher power in $1/C$ are neglected. For planetary motion in the solar system the $1/C^2$ terms are smaller than Newton's acceleration term by at least the factor 10^{-8}, but some of the consequences of these correction terms are measureable and important tests of Einstein's theory. It should be pointed out that prediction of new observable gravitational effects requires particular care; Einstein's pioneer work in gravity has shown that gravitational fields affect the basic measuring instruments of experimental physics—clocks, rulers, light rays—with which any experimental result in physics is established. Some of these effects are listed below:

1. The rate that clocks run is reduced by proximity of massive bodies; *i.e.*, clocks near the Sun will run slowly compared with identical clocks farther away from it.

2. In the presence of gravitational fields the spatial structure of physical objects is no longer describable precisely by Euclidean geometry; for example, in the arrangement of three rigid rulers to form a triangle, the sum of the subtended angle's will not equal 180°. A more general type of geometry, Riemannian geometry, seems required to describe the spatial structure of matter in the presence of gravitational fields (see PHYSICAL THEORIES, MATHEMATICAL ASPECTS OF).

3. Light rays do not travel in straight lines, the rays being deflected by gravitational fields. To distant observers the light-propagation speed is observed to be reduced near massive bodies.

Gravitational fields and general theory of relativity. In Einstein's general theory of relativity the physical consequences of gravitational fields are stated in the following way. Space-time is a four-dimensional non-Euclidean continuum, the curvature of space-time's Riemannian geometry being produced by or related to the world's matter distribution. Particles and light rays travel along the geodesics (shortest paths) of this four-dimensional geometrical world.

Null experiments and post-Newtonian effects

The experimental foundations for modern theories of gravity can be classified into two categories—null experiments and the detection of extremely small (post-Newtonian) effects. Null experiments establish the absence of conceptually possible gravitational effects, usually thereby greatly restricting the class of acceptable laws of gravity. The small differences from Newtonian gravitation, and their interpretation, are discussed below.

At the turn of the century the Hungarian physicist Baron Lóránt (Roland) Eötvös found that different materials accelerated in the Earth's field at identical rates to an accuracy of one part in 10^9. Recent experiments have increased the observed equality of accelerations to one part in 10^{11}. Newtonian theory is in accord with these results because of his postulate that gravitational force is proportional to a body's mass.

Inertial mass is a mass parameter giving the inertial resistance to acceleration of the body when responding to all types of force. Gravitational mass is determined by the strength of the gravitational force experienced by the body when in the gravitational field *g*. The Eötvös experiments, therefore, show the equality of gravitational and inertial mass for different substances.

Einstein's special theory of relativity views inertial mass as a manifestation of all the forms of energy in a body according to his fundamental relationship $E = mc^2$, E being the total energy content of a body, m the inertial mass of the body, and c the speed of light. Viewing gravitation, then, as a field phenomenon, the null result of the Eötvös experiments indicates that all forms of nongravitational energy must identically couple to or interact with the gravitational field, because the various materials in nature possess different fractional amounts of nuclear, electrical, magnetic, and kinetic energies, yet they accelerate at identical rates.

In the general theory of relativity the gravitational field also interacts with gravitational energy in the same manner as with other forms of energy, an example of that theory's universality not possessed by most other theories of gravitation. Eötvös' experiments using celestial bodies that contain a detectable fraction of internal gravitational energy were testing this feature of the general theory of relativity in the 1970s; these experiments are intended to determine whether the various solar-system bodies accelerate at identical rates in the Sun's field. Measurements of great precision, using radar and laser ranging, of the time-dependent interbody distances between the Earth and Moon, the Earth and Jupiter, and other such relationships are now possible (see below).

Gravitational consequences of the equivalence principle. A decade before he completed his full mathematical theory of gravity, Einstein predicted new gravitational effects using the equivalence principle. Observing that the equality of gravitational and inertial mass made impossible a distinction between uniform gravitational fields and accelerated coordinate systems, he proposed the null principle: no experiment can distinguish between local gravitational fields and accelerated coordinate systems. He then was able to show that clocks would run slower when near massive bodies and that light would be deflected toward massive bodies by their gravitational field.

Einstein's null principle

Newton's third dynamical law states that every force implies an equal and opposite reaction force. Modern field theories of force contain this principle by requiring every entity that is acted upon by a field to be also a source of the field. A recent null experiment established to a one-part-in-20,000 accuracy that different materials produce gravitational fields with a strength the same as they are acted upon by gravitational fields. In this experiment a sphere of solid material was moved through a liquid of identical weight density. The absence of a gravitational effect on a nearby Cavendish balance instrument during the sphere's motion is interpreted as showing that the two materials had equal potency in producing a local gravitational-field anomaly.

Other experiments have brought confirmation of Einstein's predictions to an accuracy of a few percent. Using the Mössbauer effect to monitor the nuclear reabsorption of resonant gamma radiation (hard X-rays), a shift of wavelength of the radiation which travelled vertically tens of metres in the Earth's gravitational field was measured, the slowing of clocks (in this case the nuclear vibrations are clocks) as predicted by Einstein has been confirmed to 1 percent precision. If ν and $\Delta\nu$ are clock frequency and change of frequency, respectively, h is the height difference between clocks in the gravitational field g. This is

$$\frac{\Delta\nu}{\nu} = -\frac{gh}{c^2}.$$

For a height of ten metres (about 30 feet) this effect produces only a one-part-in-10^{15} change in clock rates. The predicted deflection of light in gravitational fields was first detected during a 1919 solar-eclipse experiment. Recently progress has been made in measuring a related effect, the slowing of light's speed of propagation when near massive bodies. Timing the round-trip travel time for radar pulses between Earth and other inner planets or artificial satellites passing behind the Sun, experiments have confirmed to about 4 percent the prediction of an additional time delay, Δt, given by a formula in which M_S is the Sun's mass, R_1 and R_2 are the distances from the Sun to Earth and to the other reflecting body, D is the distance of closest approach to the Sun of the radar pulses. The additional time delay Δt is expressed as $4G$ times the Sun's mass over the cube of the velocity of light, c, times the logarithm of the quantity: four times the product of R_1 and R_2 divided by the square of D. In symbols,

$$\Delta t = \frac{4GM_s}{c^3} \ln\frac{4R_1R_2}{D^2}.$$

These radar ranging experiments to bodies in the solar system naturally complement the classical astronomical optical measurements of bodies' angular positions as seen from Earth. Radar tracking of the inner planets confirms to a few percent the precessional motion of the planets' elliptical orbits predicted by the general theory of relativity.

SOME ASTRONOMICAL ASPECTS OF GRAVITATION

Recent astrophysical discoveries such as quasi-stellar objects, pulsars, and the apparent gravitational radiation pulses being received from our galactic centre indicate that unusual astrophysical objects containing intense gravitational fields may exist in the universe. Modern physical theory predicts that a sufficiently massive object must, upon exhausting its nuclear fuel, inevitably collapse under its gravitational self-attraction, in most cases expelling part of its material in a supernovae explosion. Making only very general assumptions about the interaction properties of matter, it is concluded that the core of these collapsed stars will end either in a new superdense state of matter—the neutron star of typical density 10^{17} kilograms per cubic metre—or it is believed the core will collapse indefinitely toward a singularity, altering the properties of the surrounding space because of the enormously strong gravitational fields produced, so that physical communication with the outside world is quickly cut off, even light rays being trapped within this gravitational or black hole.

Pulsars

It is presently thought that the pulsars discovered in 1968 are neutron stars, having masses comparable to the Sun but diameters of only ten to 100 kilometres (six to 60 miles). The highly periodic electromagnetic radiation pulses coming from known pulsars with periods of from 1/30 of a second up to about a second require something like very small, but massive, rotating neutron stars as their sources.

The search for black holes

A black hole is the name given to the volume surrounding a collapsed star (an enormous mass in a tiny space), in which the gravitational field is so large that no radiation can get out; as a result it cannot be observed from outside. The astronomical search for a black hole is made difficult because of the very short time required for their formation (fractions of a second) and the relative infrequency of their creation. After being formed they emit no radiation or signals for the astronomer to detect. Present efforts are concentrated on the possibility of detecting a gravitational hole with a visible companion binary star. The gravitational wave pulses believed to be detected by the hundreds per year may be the gravitational radiation that would result from massive bodies being strongly accelerated into a large black hole (of thousands of solar masses) located near the centre of the Galaxy.

This field is experiencing such rapid theoretical and experimental development that these present viewpoints must certainly be considered as somewhat tentative and temporary. (K.L.N.)

THE ACCELERATION OF GRAVITY ON THE EARTH'S SURFACE

More than 300 years ago Galileo, in studying how things fall toward the Earth, discovered that the motion is one of constant acceleration. He was able to show that the distance a falling body travels from rest in this way varies as the square of the time. The acceleration due to gravity at the surface of the Earth is about 980 centimetres (about 32 feet) per second per second; that is, following its release, an object will gain in speed 980 centimetres per second for each second it falls.

Perhaps Galileo's most noted conjecture was that in a perfect vacuum all bodies would fall at the same rate; if they are released together, they will strike the ground together. This conjecture, in combination with his observation that the motion of free fall is one of constant acceleration, leads to the result that all bodies fall with the same acceleration. This assumption of a common acceleration has proved to be one of the cornerstones of gravitational physics; it has been tested many times—most notably in Eötvös experiments and in the recent extraordinarily precise experiments of the U.S. physicist Robert H. Dicke and his colleagues. Common acceleration indicates that a body's weight (the Earth's gravitational pull on the body) is proportional to its inertial mass—that is, to the resistance it offers to an accelerating force. The constant of proportionality is simply the acceleration of gravity. Thus, g plays a dual role: the value of the acceleration observed in free fall (the acceleration of gravity) is also the constant of proportionality between mass and weight (the force of gravity per unit mass).

Importance of g in many fields

Knowledge of the acceleration due to gravity is of importance to several different disciplines of the physical sciences. Its absolute value provides a base that, together with the standard of mass, establishes the derived standard of force. The standard of force, in turn, is a necessary quantity in the assignment of values to the electrical units of current and voltage. The acceleration due to gravity is also an important factor in the accurate pressure determinations needed for the thermodynamic temperature scale and the establishment of the International Practical Temperature Scale. Absolute measurements of the acceleration due to gravity are also of interest to the science of geodesy. Rapid advances have recently been made in setting up a world gravity network to establish reliable gravity values at a large number of base points located strategically over the Earth.

Variations in g. *Changes due to location.* Though often thought of as a constant over the surface of the whole Earth, the acceleration g varies by about ½ of 1 percent with position on the Earth's surface, from about 978 cm/sec^2 at the Equator to approximately 983 cm/sec^2 at the Earth's poles. This variation stems chiefly from the rotation of the Earth, as part of the Earth's pull is balanced by keeping objects rotating with the Earth instead of flying tangentially off into space (as mud does off a spinning wheel). This effect is also responsible for the bulge of the Earth at the Equator and the slight flattening at the poles. The distance to the centre of the Earth, therefore, increases with the bulge from the poles to the Equator, and consequently g, which for a spherical Earth of radius r_E and mass M_E is given simply by GM_E/r_E^2, is less toward the Equator.

In addition to this broad-scale variation, local variations of a few parts in 10^6 or smaller are caused by variations in the density of the Earth's crust as well as height above sea level. Further, at any one particular place, g, considered to be the resultant force, varies with time as a result of the changing gravitational attraction of the Sun and the Moon.

Changes with time. For most purposes, only knowledge of the variation of gravity with time at a fixed place (tidal variation; see TIDES) or of the gravity differences from place to place is required. Accordingly, the great bulk of gravity measurements that have been made are relative; that is, they measure only the differences between gravity values at various places. In tribute to Galileo the unit gal (equivalent to 1 cm/sec^2) has been adopted as the unit of acceleration of gravity, and this has been subdivided to milligal, essentially one part in 10^6 of g, for convenience; thus, one milligal (mgal) = 0.001 gal = 0.001 cm/sec^2.

Great progress was made during the 1960s in the development of new standards of accuracy for the measurement of g on land, on ships, and in the air, as well as in space. Recent progress in gravimetry has been influenced by improvement in the accuracy and reliability of absolute gravity determinations. As a result, a worldwide network of gravity base stations and calibration lines has been established. The methods used for the measurement of the acceleration due to gravity in terms of the fundamental units of length and time are described below.

Only two basic methods have been used to measure the absolute acceleration of gravity: by timing (1) freely falling bodies and (2) those that move under gravity but whose motion is constrained in some way, as in the case of a pendulum.

Pendulum measurements of g. Pendulum measurements of g are familiar to almost everyone who has taken an introductory course in physics. A pendulum is called simple if it consists of a heavy bob at the end of a nearly weightless arm or compound if the weight is distributed also through the arm. In the case of a simple pendulum, the time of swing is proportional to the square root of the length divided by the acceleration of gravity. This relative ease of timing of a pendulum swing offers a distinct timing advantage over free-fall measurements. The constraint on the body's free fall permits a large number of "drops" to be made very conveniently during a measured interval of time. The price one pays for this gain is that, having introduced a constraint, the effects of the constraint on the performance of the pendulum must be taken precisely into account. The English physicist Henry Kater showed (1817) that if the period of swing was the same about each of the points of support for a pendulum that could be hung from either of two fixed points, the distance separating these points of suspension was equal to the length of a simple pendulum having the same period. Once the equality of periods has been established (by adjusting the position of attached weights), the problem is then reduced to that of measurement of the common period and of the distance separating the two supports. Reversible pendulums, which can reach an ultimate accuracy in the range of a few parts in 10^6 (1 mgal), provided the only practical basis for absolute measurements of g from Kater's time until the middle of the 20th century.

The work of Henry Kater

The most accurate and most straightforward way of measuring the acceleration due to gravity is now to measure directly the acceleration of a freely moving body. Only relatively recently has it been possible by electronic control to realize the necessary accuracy in the measurement of short time intervals to permit effective measurement of g by direct free fall.

Accuracy attained in the known value of g. The early free-fall experiments (dating from 1952) used geometrical optics to define the position of an object as it fell. From 1963, direct interferometric methods using corner cube mirrors, one of which was dropped, have led to more accurate distance measurements during the free fall. More recently still, lasers have been used as light sources in free-fall interferometric devices. By the 1970s the best absolute-gravity experiments had demonstrated accuracies in the 0.01–0.05 mgal range. Furthermore, measurements with a semi-portable laser interferometer apparatus have now been made at a number of stations covering a gravity range of 4,800 mgal with an absolute accuracy of five parts in 10^8 (0.05 mgal).

For most purposes only a knowledge of the variation in gravity from place to place is required. Accordingly the great bulk of gravity measurements are relative and give gravity differences between places on the Earth. These can then be referred to an absolute system to produce gravitational values for the sites.

Accuracy of pendulum measurements. Since the time of Newton, measurements of gravity differences (strictly of the ratio of one value to another) have been made by timing the same pendulum at two sites where gravity is to be compared. Already in 1818, such relative measurements reached an accuracy of a few parts in 10^6. The most accurate work is now done with two pendulums swinging in opposite phase; in this way the sway of the support due to the reaction of the pendulums is eliminated, and also any movements of the support will produce equal and opposite changes in the periods of the two pendulums, which can be made to cancel out if the mean period of the two pendulums is used.

The accuracy of relative measurements with a pendulum depends on timing accuracy and the constancy of the conditions. Further, the difference in gravity is obtained in absolute units and therefore does not require any instrumental calibration. Accuracy of modern pendulum observations is limited by the scatter of the results when a pendulum is swung repeatedly in one place and, mainly, by changes in the pendulums that occur during transportation from place to place. The best claimed accuracy is a few tenths of a milligal.

Use of gravity meters. Up to about 1930 the pendulum was the only instrument available for relative gravity measurements, even for small scale geophysical prospecting. The development of static gravimeters restricted pendulum measurements to providing the calibration for these gravimeters. The growing number of truly absolute determinations can be expected to make even this use obsolete.

Spring gravity meters

Spring gravity meters balance the force of gravity mg on a mass m in the gravity field to be measured, against the elastic force of the spring, using either electronic or mechanical means to achieve high sensitivity. Vibrating string gravity meters in which the string's vibration frequency is determined by g have also been developed. A device of this type was employed by the Apollo 17 astronauts on the Moon to conduct a gravity survey of their lunar landing site. One of the most recent developments has been the superconducting gravimeter, an instrument in which the position of a magnetically levitated superconducting sphere is sensed to provide a measure of g.

Modern gravity meters may have sensitivities greater than 0.005 mgal, the standard deviation of observations in exploration surveys being, in the best performance, of the order of 0.01–0.02 mgal.

Differences in gravity measured with gravimeters are obtained in quite arbitrary units—divisions on a graduated dial, for example. The relation between these units and milligals can only be determined by reading the instrument at a number of points where g is known as a result of absolute or relative pendulum measurements. Further, because an instrument will not have a completely linear response, known points must cover the entire range of gravity over which the gravimeter is to be used.

Gravimetric surveys. Recently, by combining all available absolute and relative measurements, it has been possible to obtain the most probable gravity values at a large number of sites to a high degree of accuracy. The culmination of gravimetric work begun in the 1960s has been a worldwide gravity reference system having an accuracy of one part in 10^7 (0.1 mgal) or better.

Since g is an acceleration, the problem of its measurement from a vehicle that is moving and therefore unavoidably accelerating relative to the Earth raises a number of fundamental problems. Pendulum, vibrating string, and spring-gravimeter observations have been made from submarines; using gyrostabilized platforms, relative gravity measurements with accuracies approaching a few mgal have been and are being made from surface ships. Experimental measurements with various gravity sensors on fixed-wing aircraft as well as on helicopters have been carried out. Additional information about the Earth's gravitational field has been made possible through the use of artificial satellites. Tidal gravity variations are observed through the use of sensitive recording gravimeters. One of the most remarkable recent measurements was the mapping of variations in the gravitational field on the visible side of the Moon by observed perturbations in the orbits of Lunar Orbiter satellites.

The value of gravity measured at the surface of the Earth is the resultant of such component factors as (1) the gravitational attraction of the Earth as a whole, (2) centrifugal force caused by the Earth's rotation, (3) elevation, (4) unbalanced attractions caused by surface topography, (5) tidal variations, and (6) unbalanced attractions caused by irregularities in underground density distributions. Most geophysical surveys are aimed at separating out the last of these in order to interpret the geological structure. It is therefore necessary to make proper allowance for the other factors.

The free-air and Bouguer corrections factors. The first two factors imply a variation of gravity with latitude that can be calculated for an assumed shape for the Earth. The third factor, the decrease in gravity with elevation, due to increased distance from the centre of the Earth, amounts to −0.3086 mgal/m (−0.09406 mgal/ft). This value, however, assumes that material of zero density occupies the whole space between the point of observation and sea level, and it is therefore termed the free-air correction factor. In practice, the mass of rock material that occupies part or all of this space must be considered. Where the topography is reasonably flat this is usually calculated by assuming the presence of an infinite slab of thickness equal to the height of the station, h, and having an appropriate density σ; its value is $+0.04185\ \sigma h$ mgal/m or $+0.01276\ \sigma h$ mgal/ft. This is commonly called the Bouguer correction factor.

Terrain or topographic corrections can also be applied to allow for the attractions due to surface relief if the densities of surface rocks are known. Tidal effects the amplitudes of which are lower than 0.3 mgal can be calculated and allowed for.

In defining anomalies, the observed gravity g_0 is compared with the theoretical value $g\gamma$ for the latitude of the station. The difference is then corrected for the elevation, h, of the station, using the free-air correction factor, F, with or without the Bouguer correction factor, B. The topographic correction, T, is also applied, giving in symbols: free-air anomaly $= g_0 - g_\gamma + Fh + T$ and Bouguer anomaly $= g_0 - g_\gamma + (F - B)h + T$. In exploration surveys Bouguer anomalies are most commonly used. Free-air anomalies or isostatic anomalies, in which a further correction for crustal material based on the compensation of mass above a certain depth has been applied, are those generally adopted in geodetic work.

(Ja.F.)

THE GRAVITATIONAL CONSTANT, G

The gravitational constant, G, has been introduced in the first part of this article. Although G is one of the most fundamental constants in nature, it probably is the least accurately known because of the extreme weakness and universality of the gravitational interaction. The weakness is such that the force of attraction between two spheres each weighing one kilogram spaced 0.1 metre apart is only about 1.3×10^{-13} of the pull of gravity on one of the spheres. The universality of the gravitational force, already assumed by Newton, is supported by the failure of experiment to show any variation of G that depends on the kind or size of the attracting masses, their temperature, or the amount of other matter placed between them. Consequently, in order to determine G, it is not only necessary to measure very tiny forces or torques but also to do so in the presence of the much larger perturbing forces due to all of the other matter in the universe, as it is impossible to shield the masses under investigation from the rest of the universe. The classical theory of celestial mechanics is based upon Newton's law and is used to predict with great accuracy the paths of the Moon, the Sun, the planets, and other bodies through space; but solution of the mathematical equations obtained from astronomical observations does not give G uniquely, but rather—if M is the mass of one of the interacting bodies—the highly precise product MG.

Difficulties in measuring G

Principal methods of measuring G. There are three principal methods of measuring G: (1) in which the pull of the Earth is compared with that of a large natural mass, such as a mountain or other topographical mass, on that of a known mass called a test mass; (2) in which a comparison is made between the Earth's attraction and that of a known mass on a test mass, as in the common

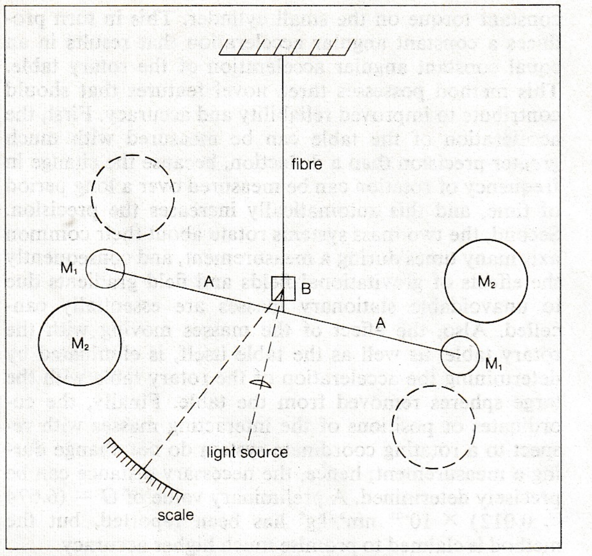

Figure 1: Modified Cavendish deflection experiment. The masses M_1 are deflected horizontally by the masses M_2, first in position shown by full circles, then at dotted circles. The twist is measured on the scale from reflection of light from a small mirror at B (see text).

chemical balance experiments; and (3) in which direct determination of the force between known masses is made in the laboratory. At the present time, experiments in the first category are of historical interest only and have not yielded a reliable value of G. As more accurate values of G become available by other methods, however, such experiments in connection with modern geophysical investigations can give valuable information concerning the densities and density gradients, especially in the Earth and Moon.

The various experimental methods of determining G in the laboratory have in the past employed some form of torsion balance or the common balance. According to the 18th-century English physicist Lord Henry Cavendish, the torsion balance was invented by the British geologist and astronomer, the reverend John Michell, for the purpose of measuring G, though Michell died before the apparatus was completed. The first reliable measurement of G was carried out by Cavendish in 1798 with a torsion balance of the Michell type. Figure 1 shows schematically the method used by Cavendish, though many details of the actual apparatus differed from those of the Figure. Two small spherical masses, M_1, are mounted on a stiff rod, A, and suspended by a torsion fibre, f. A light beam is reflected from a mirror, B, rigidly attached to A and brought to focus on a scale, S. When the large spherical masses, M_2, are placed in the fixed position shown in Figure 1, the gravitational interaction produces a torque on the small mass system, M_1, which causes A to rotate around its vertical axis. This rotation continues until the torque produced by the gravitational attraction of the small mass system, M_1, and the large mass, M_2, is balanced by the restoring torque due to the twist in the fibre. The deflection of the light beam on the scale, S, is then a measure of the twist in the fibre from which the torque, due to gravitational interaction, can be found. If the large masses, M_2, are shifted from the positions shown in the Figure into the positions indicated by the dotted circles, the small mass system is twisted in the opposite direction, and the deflection of the light beam on the scale S is reversed. The extreme smallness of the deflection of the light beam usually limits the precision of the method. This torsion-balance-deflection method was perfected by the English physicist Sir Charles Vernon Boys, who first developed the quartz-fibre suspension.

The work of Lord Henry Cavendish

The common balance method consists in supporting equal spherical masses, M_1, from the weighing pans of an equal arm chemical balance. A large spherical mass, M_2, mounted on a turntable is rotated directly under one of the masses M_1. If d is the distance between the centres of the two spheres, the downward gravitational attraction of M_2 on M_1 given by GM_1M_2/d^2 adds to the pull of gravity M_1g due to the Earth, and it tips the equal arm balance through a small angle that is measured by a pointer or optical magnification device. M_2 is then rotated until it is under the weight attached to the opposite arm of the balance, and the deflection that now takes place in the opposite direction is measured. From calibration of the balance, G can then be calculated in terms of g and d. Careful experiments were carried out with this method, principally by the English physicist John Henry Poynting, using a specially constructed balance; but M_1g was so much larger than the gravitational interaction of the two masses that the results were not as reliable as those obtained with the torsion balance, in which the pull of the Earth is not directly superposed upon the gravitational interaction.

Figure 2 shows a schematic diagram of the torsion-balance-oscillation method. The torsion balance that supports the small masses is similar to that described in Figure 1, but the large attracting masses, M_2, are placed in the same straight line that passes through the small masses, as shown in Figure 2. The torsion balance containing the small masses is then given a small displacement and the period of oscillation measured. This period can also be calculated in terms of G and the other quantities. The large masses, M_2, are then placed in the positions indicated by the dotted lines, which are in a line perpendicular to the line joining the small masses, M_1, and passing through its centre. A second determination of the period of the torsion pendulum is then measured. From these values it is possible to determine G. The advantage of this method over the torsion-balance-deflection method is that periods of oscillation can be measured with greater precision than small deflections, but it suffers from uncertainties due to the effect of gravitational gradients and neighbouring masses. This method produced what until recently was usually regarded as the most reliable value of G. In 1971 it was again used to obtain a new value of G. The new study is being carried out in the Grotta Gigante in Italy. The small masses, M_1, each weigh 10 kilograms and the large masses 500 kilograms while the torsion-wire suspension is 90 metres long. With the oscillation method, however, the torsion pendulum must be in a vacuum to prevent large damping through air resistance. The method has been modified in another study so that both the small and large mass systems are on separate torsion suspensions and are brought into resonance, but the result with this arrangement is believed to be less precise than that achieved by the U.S. physicist Paul Heyl (published in 1930; see Table).

The Table is a partial list of the measured values of G. The accuracy of the measurements made before 1930 is difficult to evaluate, but it is probable that the values re-

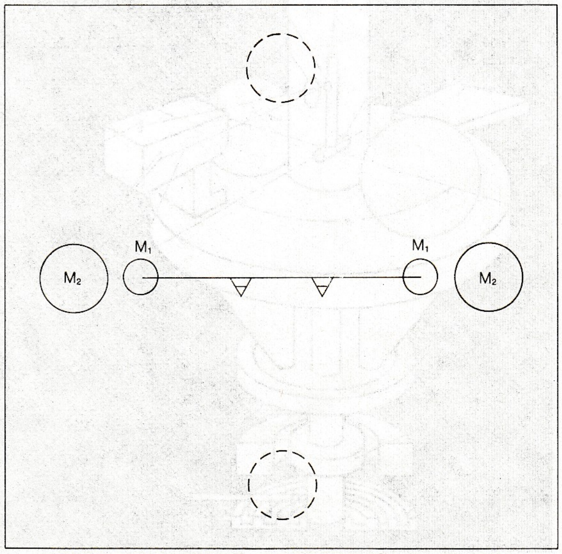

Figure 2: Elements in the torsion-balance resonance experiment (see text).

Best Values of the Gravitational Constant G
(up to 1942)

	year	method	G unit: ($10^{-11}Nm^2Kg^{-2}$)
Cavendish	1798	torsion-balance (deflection)	6.754
Reich	1838	torsion-balance (deflection)	6.61
Baily	1842	torsion-balance (deflection)	6.475
Von Jolly	1881	common balance	6.465
Wilsing	1889	metronome balance	6.596
Poynting	1891	common balance	6.698
Boys	1895	torsion-balance (deflection)	6.6576
Braun	1896	torsion-balance (oscillation)	6.6579
Eötvös	1896	torsion-balance (oscillation)	6.65
Richarz	1898	common balance	6.685
Burgess	1901	torsion-balance (deflection)	6.64
Heyl	1930	torsion-balance (oscillation)	6.670
Zahradnicek	1932	torsion-balance (resonance)	6.659
Heyl and Chrzanowski	1942	torsion-balance (oscillation)	6.673

ported by Heyl and his associates—$G = (6.670 \pm 0.015) \times 10^{-11}$ Newton-square metre per square kilogram—is the most reliable among those listed.

A new method was devised in the 1960s and is illustrated in Figure 3. Two comparatively large spherical masses (ten-kilogram tungsten spheres) are mounted on a rotary table that can be turned about its vertical axis by a servo-controlled electric motor. An airtight cylindrical chamber also is rigidly mounted upon the rotary table with its vertical axis coincident with the axis of rotation. The small mass system, which consists of a small cylinder with its axis horizontal, is supported by a torsion fibre fastened to the top of the cylinder so that the fibre hangs in the axis of rotation of the table, as shown in Figure 3. The gravitational interaction between the large and small mass systems tends to rotate the small mass system in a direction that brings the axis of the small cylinder into coincidence with an imaginary line passing through the centres of mass of the large spheres. This changes the angle θ. A change in θ, however, also produces a change in the angle β between a light beam and its reflection from a mirror mounted on the small mass system. The light source and the photo-diode sensing system are rigidly mounted on the table in such a way that a change in β generates a photo-diode signal that actuates the servo-motor, which in turn rotates the table so that β and θ remain constant to less than one-half second of arc. Since the angle θ remains constant, the gravitational interaction between the small and large mass systems produces a

From an illustration by Marina Givotovsky in *Physics Today* (May, 1971)

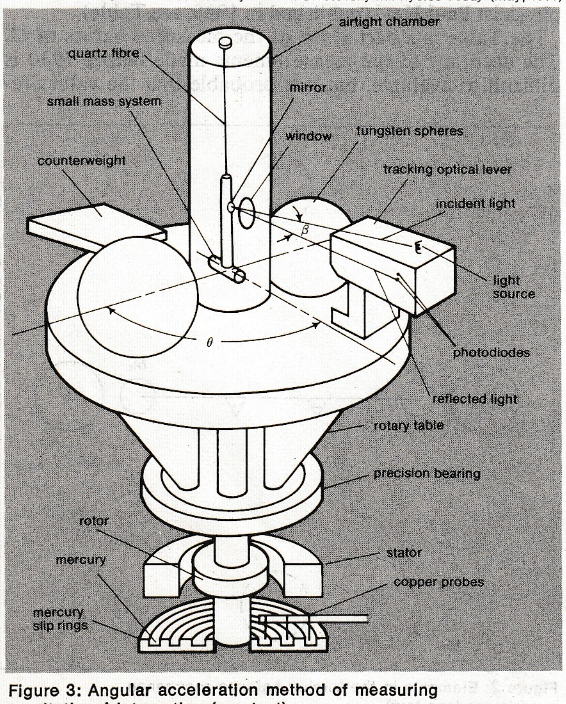

Figure 3: Angular acceleration method of measuring gravitational interaction (see text).

constant torque on the small cylinder. This in turn produces a constant angular acceleration that results in an equal constant angular acceleration of the rotary table. This method possesses three novel features that should contribute to improved reliability and accuracy. First, the acceleration of the table can be measured with much greater precision than a deflection, because the change in frequency of rotation can be measured over a long period of time, and this automatically increases the precision. Second, the two mass systems rotate about their common axis many times during a measurement, and consequently the effects of gravitational fields and field gradients due to unavoidable stationary masses are essentially cancelled. Also, the effect of the masses moving with the rotary table, as well as the table itself, is eliminated by determining the acceleration of the rotary table with the large spheres removed from the table. Finally, the coordinates or positions of the interacting masses with respect to a rotating coordinate system do not change during a measurement; hence, the necessary distance can be precisely determined. A preliminary value of $G = (6.674 \pm 0.012) \times 10^{-11}$ nm^2/kg^2 has been reported, but the method is claimed to promise much higher accuracy.

Weighing the Earth. If Newton's equation mentioned above is applied to the attraction of the Earth on a small mass, M_1, with g as the acceleration of gravity and M_E and r_E as the mass and effective radius of the Earth, respectively, then $g = GM_E/r_E$. Since g and r_E are known, the mass of the Earth, M_e, can be obtained if G is known. For this reason Cavendish and some of the early workers referred to their work as "weighing the Earth." Actually, no way is yet known for reliably obtaining the mass of the Earth, Moon, planets, or other heavenly bodies—say in kilograms, tons, etc.—without a knowledge of G. The mass of the Earth is approximately 5.98×10^{24} kilograms (13.18×10^{24} pounds). Since the radius of the Earth is known, its volume can be calculated so that the mean density of the Earth is obtained. This mean density of the Earth is approximately 5.52 times that of water, while the mean density of the Sun is 1.43 times that of water. These mean densities are becoming of considerable importance in geophysical research.

Important gravities dependent on *G*

Fundamental character of G. In addition to the above practical needs for more accurate values of G, because of its fundamental nature it necessarily must enter into all major cosmological questions. Some cosmological theories predict a minute decrease in G of about one part in 10^{10} per year. Very precise measurements made in the early 1970s of radar-echo time delays between the Earth and Mercury indicate that G does not vary by more than four parts in 10^{10} per year, unless there are unknown compensating factors in the experiments. Speculation that G may be influenced by the position in space led to proposals for the measurement of G in space vehicles. Other suggestions postulate effective changes in G at very small and at very large distances and with time. The absolute measurements of G are not yet accurate enough nor is the time base long enough to resolve these or many other important questions. On the other hand, G seems, to the limit of present accuracy, to be a truly fundamental constant, both in magnitude and sign, and independent of perturbing effects not only in the case of the gravitational interaction of matter but also that of antimatter as well.

(J.W.B.)

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(K.L.N./Ja.F./J.W.B.)

Graywackes

Graywacke is the name applied to generally dark-coloured, very strongly bonded sandstones that consist of a heterogeneous mixture of rock fragments, feldspar, and quartz of sand size (1/16–2 millimetres [0.002–0.078 inches]), together with appreciable amounts of mud matrix (less than 1/16 millimetres [0.002 inches]). Almost all graywackes originated in the sea, and many were deposited in deep water by density (turbidity) currents.

The name graywacke (from the German *Grauwacke*) was first used by the German mineralogist Abraham Werner in 1787. It describes the colour and texture of the rock, "wacke" being a term used for the heterogeneous weathering products derived from igneous and metamorphic rocks. The classic German locality for graywacke is the Devonian and Carboniferous sequence of the Harz Mountains.

In 1818 John Mawe wrote, "Geologists differ much as to what is, and what is not, Grey Wacce." This is still true. Recent definitions by various geologists characterize graywackes as sandstones that (1) contain more than 10, 15, or 20 percent mud matrix; (2) contain less than 75 percent quartz and more than 25 percent nongranitic feldspars and rock fragments; (3) have similar mineralogy and chemistry to those from the Harz Mountains; or (4) display associations of sedimentary structures that are characteristic of "turbidites"—*i.e.*, rocks deposited by turbidity currents. Many geologists now use the name only in a general sense, in reference to the dark, usually quartz-poor, often well-indurated sandstones that occur in turbidite (or flysch) sequences.

There are great differences between the Harz and Bunter sandstones in Germany, the Aberystwyth and Millstone grits in Great Britain, and the Franciscan and Navajo formations of the United States. The differences in structure and mineralogy reflect fundamental differences in both mode of deposition and relationship to phases of earth history. The persistence of the term graywacke, despite the confusion surrounding it, is testimony to the need to express these differences succinctly, for this term has been applied to the first of each pair cited above. The plethora of definitions indicates the difficulties experienced in finding generally applicable criteria that adequately express the differences.

This article treats the composition, properties, occurrence, and origin of graywacke. See SANDSTONES; SEDIMENTARY ROCKS; MARINE SEDIMENTS; and SHALES for information on related rock types; see DENSITY CURRENTS for additional discussions of origin; and see MOUNTAIN-BUILDING PROCESSES and EARTH, GEOLOGICAL HISTORY OF for the geological significance of graywackes.

PHYSICAL AND CHEMICAL PROPERTIES

Size, shape, and orientation of particles

Texture and structure. Graywackes typically are poorly sorted, and the grain sizes present range over three orders of magnitude—*e.g.*, from 2 to 2,000 microns (8×10^{-5} to 8×10^{-2} inch). Commonly, the coarsest part of a graywacke bed is its base, where pebbles may be abundant. Shale fragments, which represent lumps of mud eroded from bottom sediments by the depositing current, may be concentrated elsewhere in the bed.

Many graywackes contain much mud, typically 15–40 percent, and this increases as the mean grain size of the rock decreases. The particles forming the rock are typically angular. This, and the presence of the interstitial mud matrix, has led to these rocks being called "microbreccias." The fabric and texture indicate that the sediments were carried only a short distance and were subject to very little reworking by currents after deposition.

Although many geologists believe that the mud matrix accumulated simultaneously with the coarser material, some believe it was derived from alteration of rock fragments subsequent to burial. This question remains unresolved. Some graywackes are notably deficient in matrix (less than 10 percent, or even less than 5 percent). This deficiency can occur at the base of beds or be characteristic of beds as a whole. No detailed explanation has yet been suggested.

Sections of graywackes cut parallel to the bedding display alignment of the long axes of the grains. This usually is best developed in the lower-middle part of a bed. The alignment direction varies from level to level in the bed and commonly deviates from the direction of current markings on the underside of the bed, although some workers report parallelism between grain orientation and current markings. Imbrication (overlapping) of grains is observed in sections cut normal to the bedding. The origin of this variability in grain orientation and its deviation from current markings are not understood.

Graded bedding and interbedding

Graywacke sequences are noted for having a characteristic, and usually well-developed, association of sedimentary structures. Typically the beds are sheetlike and are interbedded in a regular fashion with shales, each bed paralleling its predecessor with almost mathematical precision (Figure 1).

The thicknesses of beds in a graywacke sequence are log-normally distributed (that is, the logarithms of bed thickness are distributed about some mean value), or nearly so, and typically there is a strong positive correlation between maximum grain size and bed thickness. Some graywacke sequences that are called "proximal turbidites," meaning that they accumulate near the source area, largely lack this parallelism and regular interstratification.

The most widespread internal structure of graywackes is graded bedding (Figure 1), although some sequences display it poorly. Sets of cross strata more than three centimetres (1.28 inches) thick are very rare, but thinner sets are very common. Parallel lamination is very common, and convolute bedding is usually present. These internal structures are arranged within graywacke beds in a regular sequence. They appear to result from the action of a single current flow and are related to changes in the hydraulics of the depositing current. In some beds, the upper part of the sequence of structures is missing, presumably because of erosion or nondeposition. In others, the lower part is missing. This has been attributed to change in the hydraulic properties of the depositing current as it moves away from its source and its velocity decreases to the point at which the first sediment deposited is laminated, rather than massive and graded as is the case closer to the source.

The most typical external structures of graywacke beds